Comment on "Calculation of electromagnetic field components for a fundamental Gaussian beam"

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In this Comment, we point out that the ninth-order expressions for electromagnetic fields for a fundamental Gaussian beam given by Wang and Webb [Phys. Rev. E **72**, 046501 (2005)] are incorrect. The correct expressions for the electromagnetic field components up to ninth order are given.

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In order to study the interaction between a single electron and a focused laser pulse, Wang and Webb [1] derived the seventh- and ninth-order expressions for electromagnetic field components of a Davis-Barton-symmetrized description of a fundamental Gaussian beam [2,3]. Here we point out that the seventh-order correction to the description has been given by Cao and co-workers [4] for studying the electron dynamics in laser accelerators, whereas Wang and Webb's ninth-order correction is incorrect.

For the convenience of readers, we recapitulate most of the equations and use the same notation as in Ref. [1]. In the Lorentz gauge, a vector potential $\mathbf{A}$ for the field of a monochromatic beam in an isotropic, homogeneous, nonmagnetic, and nonconducting medium can be defined using the Helmholtz equation

$$\nabla^2\mathbf{A} + k^2\mathbf{A} = 0, \tag{1}$$

where time dependence $e^{i\omega t}$ is assumed and $k=2\pi/\lambda$ is the wave number in the medium, with λ the wavelength. The electric and magnetic fields are given by

$$\mathbf{E} = -(i/k)\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{A}) - ik\mathbf{A},$$

$$\mathbf{H}/\sqrt{\epsilon} = \boldsymbol{\nabla}\times\mathbf{A}, \tag{2}$$

where ϵ is the permittivity of the medium. For a beam polarized in x and propagating in z directions, $\mathbf{A}$ is written as [2,3]

$$\mathbf{A} = \psi(x,y,z)e^{-ikz}\hat{\mathbf{x}} = \psi(\xi,\eta,\zeta)e^{-i\zeta/s^2}\hat{\mathbf{x}}, \tag{3}$$

where $\xi=x/r_0$, $\eta=y/r_0$, and $\zeta=z/l$, with r_0 and $l=kr_0^2$ being the beam waist radius and the diffraction length, respectively. In terms of ψ, the Helmholtz equation (1) becomes

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2} - 2i\frac{\partial}{\partial\zeta}\right)\psi = -s^2\frac{\partial^2\psi}{\partial\zeta^2}, \tag{4}$$

with $s=1/(kr_0)$ being assumed small number. By following Davis [2] and Barton and Alexander [3] the solution of Eq. (4) is expanded as a sum of powers of s^2,

$$\psi = \sum_{n=0}^{\infty} s^{2n}\psi_{2n}, \tag{5}$$

with ψ_{2n} satisfying, for $n=0,1,2,\ldots$,

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2} - 2i\frac{\partial}{\partial\zeta}\right)\psi_{2n} = -\frac{\partial^2\psi_{2n-2}}{\partial\zeta^2}, \tag{6}$$

where we have assumed $\psi_{-2}\equiv 0$ for the convenience of notation. Davis [2] solved ψ_2 up to the third order, while Barton and Alexander [3] obtained ψ_4 up to the fifth order and proposed a symmetrizing scheme. In a study of the electron dynamics Cao and co-workers [4] derived ψ_6, the seventh-order correction. Wang and Webb [1] calculated the seventh- and ninth-order corrections, and although their seventh-order correction ψ_6 is consistent with that by Cao *et al.* [4], their ninth-order term ψ_8, Eq. (15) in Ref. [1], is incorrect, as can be easily checked by substituting their ψ_8 into Eq. (6) with $n=4$. The correct ψ_8 should read

$$\psi_8 = \left(\tfrac{1}{24}q^{12}\rho^{16} + iq^{11}\rho^{14} - \tfrac{37}{6}q^{10}\rho^{12} - 4iq^9\rho^{10} - 15q^8\rho^8 + 40iq^7\rho^6 + 60q^6\rho^4 + 120q^4\psi_0,\right. \tag{7}$$

where $\rho^2=\xi^2+\eta^2$, $q=1/(i+2\zeta)$, and $\psi_0=iqe^{-iq\rho^2}$. As a result, in place of Eqs. (19) and (21) in Ref. [1], the expressions of electric field components E_x and E_z, up to ninth order and after symmetrization by Barton and Alexander's scheme [3], are given by

$$\begin{aligned}E_x = \Big\{&1 + (-2q^2\xi^2 - q^2\rho^2 + iq^3\rho^4)s^2 + \big[(8q^4\rho^2 - 2iq^5\rho^4)\xi^2 \\ &+ 2q^4\rho^4 - 3iq^5\rho^6 - \tfrac{1}{2}q^6\rho^8\big]s^4 + \big[(-30q^6\rho^4 + 12iq^7\rho^6 \\ &+ q^8\rho^8)\xi^2 - 5q^6\rho^6 + 9iq^7\rho^8 + \tfrac{5}{2}q^8\rho^{10} - \tfrac{1}{6}iq^9\rho^{12}\big]s^6 \\ &+ \big[\big(112q^8\rho^6 - 56iq^9\rho^8 - 8q^{10}\rho^{10} + \tfrac{1}{3}iq^{11}\rho^{12}\big)\xi^2 + 14q^8\rho^8 \\ &- 28iq^9\rho^{10} - 10q^{10}\rho^{12} + \tfrac{7}{6}iq^{11}\rho^{14} \\ &+ \tfrac{1}{24}q^{12}\rho^{16}\big]s^8\Big\}E_0\psi_0e^{-i\zeta/s^8},\end{aligned} \tag{8}$$

$$\begin{aligned}E_z = \Big\{&-2qs + (6q^3\rho^2 - 2iq^4\rho^4)s^3 + (-20q^5\rho^4 + 10iq^6\rho^6 \\ &+ q^7\rho^8)s^5 + \big(70q^7\rho^6 - 42iq^8\rho^8 - 7q^9\rho^{10} + \tfrac{1}{3}iq^{10}\rho^{12}\big)s^7 \\ &\times\big(-252q^9\rho^8 + 168iq^{10}\rho^{10} + 36q^{11}\rho^{12} - 3iq^{12}\rho^{14} \\ &+ \tfrac{1}{12}q^{13}\rho^{16}\big)s^9\Big\}\xi E_0\psi_0e^{-i\zeta/s^8},\end{aligned} \tag{9}$$

where E_0 denotes the electric field amplitude at beam's focal point. Here we do not give an expression for E_y since it is the same as in Eq. (20) of Ref. [1]. Notice that the difference lies only in the s^8 term and s^9 term in E_x and E_z, respectively, since the discrepancy in the vector potential occurs for ψ_8 only. The magnetic field components can be obtained by $H_x/\sqrt{\epsilon}=E_y$, $H_y/\sqrt{\epsilon}=E_x(\xi\leftrightarrow\eta)$, and $H_z/\sqrt{\epsilon}=E_z(\xi\leftrightarrow\eta)$, where $\xi\leftrightarrow\eta$ denotes interchanging ξ and η. The beam power, which can be defined by the time-averaged energy flux crossing the $z=0$ plane, is given by

$$P=\frac{1}{2}\iint \mathrm{Re}[\mathbf{E}\times\mathbf{H}^*]\cdot d\boldsymbol{\sigma}=\frac{\sqrt{\epsilon}|E_0|^2\pi r_0^2}{4}\left(1+s^2+\frac{3}{2}s^4+3s^6+\frac{15}{2}s^8\right) \tag{10}$$

in place of Eq. (22) in Ref. [1]. It is noted that because of the incorrect ninth-order correction term, the significant improvement in accuracy claimed by the authors is not true, as can be seen from Table I for $s=0.30$ and $s=0.40$ in Ref. [1]. In addition, the sudden increase of the coefficient of s^8 in Eq. (22) for the energy flux claimed in Ref. [1] is not correct either.

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